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Insurance and Corporate Risk Management

Richard D. MacMinn*

Abstract

This paper provides a model of a competitive financial market economy in which there are not only stock and bond markets but also insurance markets. In the model's most naive form, the analysis shows that the corporate value of the insured firm equals that of the uninsured firm and in this case the firm has no active role to assume in managing corporate risk. The model is then modified to incorporate costly bankruptcy and agency problems. Then the analysis shows that the corporation has an incentive to purchase insurance because it may eliminate or reduce the bankruptcy and/or agency costs.

Introduction

Insurance contracts are regularly purchased by corporations and play an important role in the management of corporate risk. This role has not been analyzed in finance even though insurance contracts are simply another type of financial contract in the nexus of contracts which is the corporation. The purpose of this paper is to provide a model which is robust enough to allow for an investigation of the scope of insurance contracts in the management of corporate risk.

In the insurance literature, the incentive to buy insurance is often assumed to be risk aversion. Risk aversion is a sufficient motivation for the closely held corporation but, as Mayers and Smith observe, not for the publicly held corporation. Mayers and Smith claim that "The corporate form provides an effective hedge since stockholders can eliminate insurable risk through diversification," [10, p. 282]. An equivalent claim is that the value of the insured corporation is the same as the value of the uninsured corporation. If

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this claim holds, then insurance is not a necessary tool in managing corporate risk. A characterization of the market economies in which the claim does and does not hold should be important to corporate managers as well as insurance companies. Apparently no model in the literature exists which addresses this claim.[1] The purpose of this paper is, first, to characterize the corporate and market environment in which insurance is irrelevant (i.e., establish the claim) and, second, to modify the model in order to characterize the corporate and market environments in which insurance is an important tool in managing corporate risk.

The first step is to establish the claim that corporations need not buy insurance since competitive risk markets already provide sufficient opportunity to diversify corporate risk. To establish or refute this claim requires a model of the economy which includes a stock market as well as an insurance market. Such a market model is provided in the Basic Model section of this paper. The basic model includes two types of financial contracts and two sources of uncertainty. The analysis demonstrates that any insurance decision made by the corporation may be reversed by any individual on personal account. Equivalently, a no arbitrage condition guarantees that stock and insurance contracts must be priced the same; then it is a simple matter to show that the net present value of the insurance decision is zero. Next, the basic model is extended so that it incorporates risky debt contracts as well as stock and insurance contracts. The analysis shows that as long as bankruptcy is costless the same reasoning applies. Since the value of the corporation is the sum of the values of its financial contracts and the net present value of the insurance is zero, the claim is established.

In the section, Costly Bankruptcy, the model is altered to allow for the transaction costs of bankruptcy. Mayers and Smith [10] note that bankruptcy costs provide the firm with an incentive to insure because, by shifting risk to the insurance company, the firm decreases the probability that the cost is actually incurred. The analysis here shows that the total market value of the insured firm is equal to the total market value of the uninsured firm plus the present value of the savings on bankruptcy costs; equivalently, the analysis shows that the value of the insured firm exceeds the value of the uninsured firm. This would seem to provide the corporation with an incentive. However, it is also shown that insurance increases the value of the firm's debt but that it decreases the value of the firm's equity. The firm maximizing stock market value does not have an incentive to insure.

In the section, Agency Problems, the basic model is generalized so that it incorporates the conflicts of interests between corporate management and bondholders. Conflict of interest problems arise when the corporate manager, acting in the interests of stockholders, has the incentive to select actions which are not fully consistent with the interests of bondholders. Two classic examples of the conflict of interest problem are developed. Then the analysis necessary to show how the insurance contract may be used to limit the divergence between the interests of bondholders and management is developed. The first agency conflict considered is usually referred to as the under investment problem. In this example, the manager of a levered firm has an incentive to limit the scale of investment because the additional returns from further investment accrue primarily to bondholders. Although Mayers and Smith do not discuss this conflict of interests problem, one should expect the insurance contract to play an important role in limiting the divergence of interests between the two groups. The analysis here shows that an appropriately selected insurance portfolio will increase the safety of debt and allow stockholders to capture all the additional returns from further investment. In addition, the corporation may increase both the value of its debt and the value of its stock by selecting appropriate insurance coverage. The second agency conflict considered is usually referred to as the asset substitution problem, or equivalently, as the risk shifting problem. Once a corporation has obtained debt financing, it is well known that by switching from a relatively safe investment project to a riskier one, the corporation can increase the value of its equity at the expense of its bondholders. Mayers and Smith discuss this conflict and note that rational bondholders recognize this incentive to switch and incorporate it into the bond price. As a result, an agency cost is represented in the bond price and a reduction in the corporate value. Mayers and Smith also note that one role insurance plays, in this corporate environment, is in bonding the corporation's investment decision. They suggest that the incentive to include insurance covenants in bond contracts increases with firm leverage. The analysis here shows that the asset substitution problem only exists for highly levered firms and that an indenture provision, requiring insurance, can be structured so that any incentive for risk shifting is eliminated. Thus, the model shows how insurance may be used to eliminate this agency cost. The final section of this paper presents some conclusions and comments on the role which insurance contracts play in managing corporate risk.

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[1] The claim is certainly intuitively appealing. If the corporation is viewed as a set of financial contracts, then establishing this claim will provide a generalization of the 1958 Modigliani-Miller Theorem. The generalized theorem would say that the composition of the contract set is irrelevant. It may also be noted that this generalized theorem is implied by the Coase Theorem [3].

The Basic Model

Assume that there are many individuals indexed by i in the set I and that there are many firms indexed by f in the set F. There are two dates, "now" and "then". All decisions are made now and all payoffs from those decisions are received then. The payoffs depend on which state of nature ω in the set Ω occurs then. First, the model is developed with two financial markets: a stock market and an insurance market. Second, the model is generalized to incorporate three financial markets: the stock market, the insurance market, and the bond market. The Fisher model is used in this setting.[2]

The Stock Market and the Insurance Market

Individual i is endowed with income now and then represented by the pair (m_{i0}, m_{i1}). The individual has a consumption pair (c_{i0}, c_{i1}) and an increasing concave utility function $u_i : D \rightarrow \mathbf{R}$, where D is a subset of $\mathbf{R}^2$, which expresses the individual's preferences for consumption now versus then. In order to introduce uncertainty, let $(\Omega, \mathbf{B}, \Psi)$ denote the probability space for individual i, where Ω is the set of states of nature, $\mathbf{B}$ is the event space, and Ψ is the probability measure. Suppose there are only a finite number of states of nature (i.e., $\Omega = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_N\}$); in this case, the event space $\mathbf{B}$ is the power set (i.e., the set of all subsets of Ω). To make the uncertainty operational suppose that the consumer can only transfer dollars between periods by buying or selling stock and/or by selling insurance.[3] In this complete markets setting, suppose that a stock of type ω is a promise to pay one dollar if state ω occurs and zero otherwise; similarly, suppose an insurance contract of type ω is a promise to pay one dollar against a loss in state ω and zero otherwise. Let $p(\omega)$ be the price of the stock which yields one dollar in state ω and zero otherwise and let $x_i(\omega)$ be the number of state ω shares purchased. Similarly, let $q(\omega)$ be the price of insurance against a loss in state ω and let $y_i(\omega)$ be the number of insurance contracts. Then the individual's consumption now and then are

$$c_{i0} = m_{i0} - \Sigma_\Omega[p(\omega)\, x_i(\omega) + q(\omega)\, y_i(\omega)] \tag{0}$$

and

$$c_{i1}(\omega) = m_{i1} + x_i(\omega) + y_i(\omega), \tag{1}$$

respectively. Now the problem can be stated as[4]

$$\text{maximize } \textstyle\int_\Omega u_i(c_{i0}, c_{i1})\, d\Psi,$$

$$\text{subject to } y_i \leq 0$$

where c_{i0} and c_{i1} are defined by (0) and (1), respectively. Since the number of states of nature is finite, observe that the expected utility is

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[2] See [4]. The Fisher model is developed under uncertainty in [6].

[3] Individual investors are not assumed to have insurable losses and so are not allowed to purchase insurance contracts. The losses which are introduced below are corporate losses.

[4] In its standard form the Fisher problem is stated as a constrained maximization problem in which the consumer selects the consumption now and then to maximize expected utility subject to the budget constraint, i.e.,

$$\text{maximize } \textstyle\int_\Omega u_i(c_{i0}, c_{i1})\, d\psi$$

$$\text{subject to } c_{i0} + \Sigma_\Omega\, p(\omega)c_{i1}(\omega) = m_{i0} + \Sigma_\omega\, p(\omega)m_{i1}.$$

The problem stated in (P) above incorporates this budget constraint.

$$\int_\Omega u_i(c_{i0}, c_{i1})\, d\Psi = \Sigma_\Omega\, u_i(c_{i0}, c_{i1}(\omega))\, \psi(\omega),$$

where $\psi(\omega)$ is the probability that ω state occurs. It follows that the consumer's problem may be rewritten as

$$\text{maximize } \Sigma_\Omega\, u_i(c_{i0}, c_{i1})\, d\Psi,$$

$$\text{subject to } y_i \leq 0 \tag{P}$$

Since there is no constraint on the purchase or sale of stock, the first order conditions in the stock market are of the form:

$$-\Sigma_\Omega D_1 u_i(c_{i0}, c_{i1}(\omega))\, p(\omega)\, \psi(\omega) + D_2 u_i(c_{i0}, c_{i1}(\omega))\, \psi(\omega) = 0,^5$$

or equivalently,

$$p(\omega) = \frac{D_2 u_i(c_{i0}, c_{i1}(\omega))\, \psi(\omega)}{\Sigma_\Omega\, D_1 u_i(c_{i0}, c_{i1}(\omega))\, \psi(\omega)} \tag{2}$$

for all $\omega \in \Omega$. The right-hand side is simply the marginal rate of substitution (i.e., consumption now for consumption in state ω), so this is the standard condition that price equals the marginal rate of substitution.

Next, consider the conditions under which the consumer will supply contracts in the insurance market. The consumer will want to supply (i.e., take a short position in) an insurance contract against state ω if expected marginal utility is positive. The expected marginal utility of the short position is:

$$\Sigma_\Omega\, D_1 u_i(c_{i0}, c_{i1}(\omega))\, q(\omega)\, \psi(\omega) - D_2 u_i(c_{i0}, c_{i1}(\omega))\, \psi(\omega) \geq 0$$

if and only if $q(\omega) \geq p(\omega)$ by (2). If the price of the insurance contract is strictly greater than the price of the stock contract then there is an arbitrage opportunity since by going long in the stock and short in the insurance the consumer may increase consumption now without decreasing consumption then. Therefore, if $q(\omega) > p(\omega)$, the supply of insurance contracts will be indefinitely large. Of course, if $q(\omega) < p(\omega)$, the consumer will not supply any insurance contracts in this state. Hence, letting $y(\omega) \equiv \Sigma_i\, y_i(\omega)$, this analysis yields the market supply of insurance contracts shown in figure one.

Next, consider the value of the firm. The firm has gross earnings $R_f(\omega)$ and is subject to insurable losses $L_f(\omega)$.[6] Suppose that the firm's gross earnings

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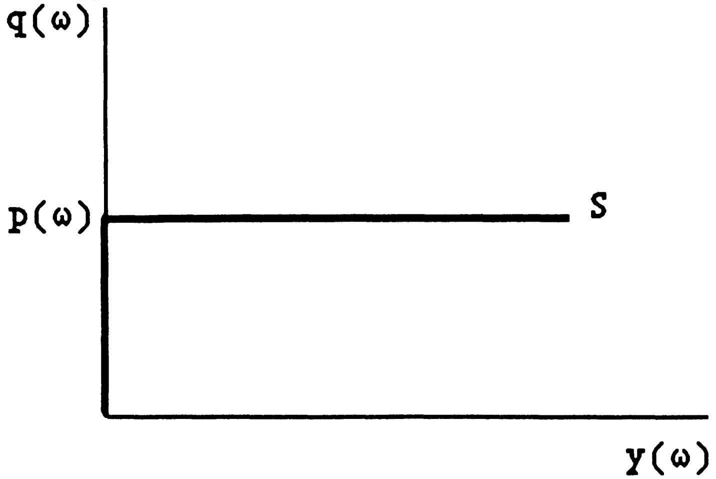
[5] The partial derivative of u_i with respect to its first argument c_{i0} will be denoted by D_1u_i. Similarly, the partial derivative of u_i with respect to its second argument will be denoted by D_2u_i, and the second partial derivative of a function, such as u_i, with respect to its jth argument will be denoted by $D_{jj}u_i$. This notation for derivatives will be used throughout the paper.

[6] The notion of insurable losses is used as a primitive (i.e., undefined) concept here. By implication there are also uninsurable losses. The uninsurable losses are incorporated by making corporate earning R_f a random variable.

Figure 1: Supply in the Insurance Market



function is increasing and the insurable loss function is decreasing in the state of nature. Then the net earnings function of the uninsured firm is $\Pi_f(\omega) = R_f(\omega) - L_f(\omega)$ and it is increasing in the state of nature. Although the firm is liable without limit, its stockholders are not. Therefore, the payoff to stockholders in this uninsured firm is $\max\{\Pi_f(\omega), 0\}$. Let E^U denote the event that the net corporate earnings, of the uninsured firm, are negative. Then the payoff to equityholders may also be expressed as $\Pi_f(\omega)$ if $\omega \in \Omega - E^U$ and zero if $\omega \in E^U = \{\omega \in \Omega \mid \Pi_f(\omega) < 0\}$.[7] E^U is simply the event that the firm is insolvent. In this complete market setting, the stock market value of the uninsured firm is equivalent to the value of a portfolio with state payoffs $\max\{0, \Pi_f(\omega)\}$. Therfore, letting $S_f{}^U$ denote the stock market value of the uninsured firm, one obtains

$$S_f{}^U = \Sigma_\Omega\, p(\omega) \max(0, \Pi_f(\omega)) = \Sigma_{\Omega - E^U}\, p(\omega)\, \Pi_f(\omega). \tag{3}$$

The firm may insure against losses in any state of nature by purchasing insurance contracts at prices $q(\omega)$. If the firm does purchase insurance then its net earnings in any state become $\Pi_f(\omega) + y_f(\omega) = R_f(\omega) - L_f(\omega) + y_f(\omega)$; the firm may purchase complete insurance by setting $y_f(\omega) = L_f(\omega)$ for all $\omega \in \Omega$. Let E^I denote the event that the insured firm is insolvent and let $\Omega - E^I = \{\omega \in \Omega \mid \Pi_f(\omega) + y_f(\omega) \geq 0\}$ denote its complement. Observe that if $R_f(\omega') > 0$ and $\Pi_f(\omega') < 0$ for some state $\omega' \in \Omega$ then a coverage level $0 < y_f(\omega') \leq L_f(\omega')$ exists such that, although $\omega' \in E^U$, $\omega' \in \Omega - E^I$. Equivalently, the existence of a state ω' implies that, for appropriate coverage levels, the insolvency set of the insured firm is a subset of the insolvency set of the uninsured firm (i.e., $E^I \subset E^U$). Also observe that once the insolvency set E^I is determined, no value maximizing firm will choose to purchase insurance against states in the insolvency set since that action will not create value for the firm's equityholders. It follows that the stock market value of the insured firm is S_f^I where

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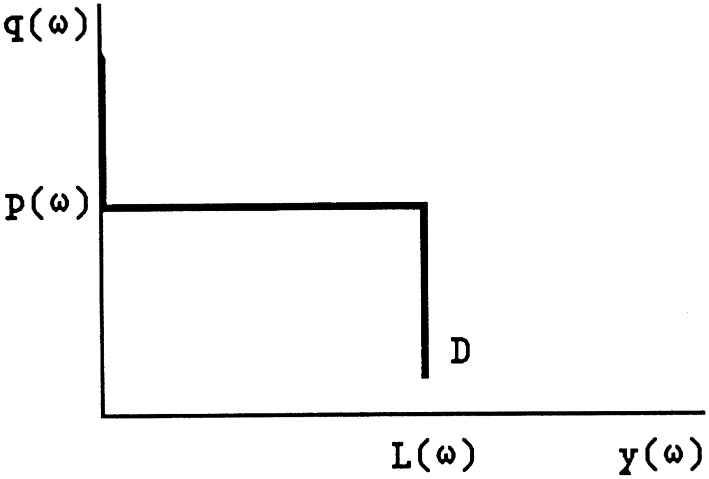
[7] The event $\Omega - E^U$ is the complement of the event E^U, or equivalently, the set of states of nature in Ω which do not belong to E^U.

$$S_f^I = \Sigma_\Omega\, p(\omega) \max\{0, \Pi_f(\omega) + y_f(\omega)\} - \Sigma_{\Omega - E^I}\, q(\omega)\, y_f(\omega)^8$$

$$= \Sigma_{\Omega - E^I}\, p(\omega)\, [\Pi_f(\omega) + y_f(\omega)] - q(\omega)\, y_f(\omega). \tag{4}$$

Differentiating the stock value with respect to $y_f(\omega)$ yields $[p(\omega) - q(\omega)]$. If this difference in prices is negative then the firm will not insure, while if this difference is positive then the firm will select to fully insure (i.e., set $y_f(\omega) = L_f(\omega)$). Of course, if the price difference is zero then the firm is indifferent between insuring and not insuring. Hence, letting $L(\omega) = \Sigma_f L_f(\omega)$ denote the aggregate insurable losses in state ω, this analysis yields the demand for insurance shown in figure two.

Figure 2: Demand in the Insurance Market



Collecting the results on demand and supply yields the equilibrium positions shown in figure three. The equilibrium price for the insurance contract is obviously $p(\omega)$; as a result, the stock value of the insured firm in (4) may be rewritten as

$$S_f^I = \Sigma_{\Omega - E^I}\, p(\omega)\, \Pi_f(\omega). \tag{5}$$

The choice of coverage levels for each state determines the solvency set $\Omega - E^I$ but the form of equation (5) shows that if the firm selects different coverage levels which do not change the solvency set then the stock value of the firm will be unaltered. What is more, note that the positions of the market

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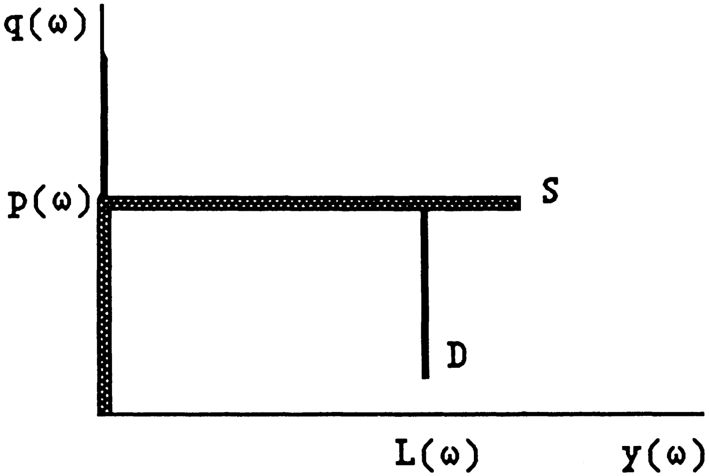
[8] If the firm were to insure in insolvency states then the stock value would be .

$$\Sigma_{\Omega - E^I}\, p(\omega)\, \Pi_f(\omega) - \Sigma_{E^I}\, q(\omega)\, y_f(\omega) < S_f^I.$$

Figure 3: Insurance Market Equilibria



equilibria show that the insurance coverage in the solvency states is not only a matter of indifference to the firm but also to the market.

Although the firm is indifferent to various coverage levels in the insurance market, it may not be indifferent to the insurance decision which determines the solvency set. Using (4) and (5), note that the difference in stock values is

$$\begin{aligned} S_f^I - S_f^U &= \Sigma_{\Omega - E^I}\, p(\omega)\, \Pi_f(\omega) - \Sigma_{\Omega - E^U}\, p(\omega)\, \Pi_f(\omega) \\ &= \Sigma_{E^U - E^I}\, p(\omega)\, \Pi_f(\omega) \leq 0. \end{aligned} \tag{6}$$

If the insolvency sets E^U and E^I are the same then the stock market value of the insured firm equals the stock market value of the uninsured firm. If it is possible for the firm to reduce the size of its insolvency set by insuring (i.e., make $E^I \subset E^U$), then the stock market value of the insured firm is less than the stock market value of the uninsured firm because $\Pi_f(\omega) < 0$ for all $\omega \in E^U - E^I$. This result may be interpreted by noting that although the firm is able to remain solvent in more states (i.e., $E^U - E^I$) solvency simply allows the firm to payoff on other explicit or implicit contracts; since the insurance decision has a zero net present value, the stockholders are left with a capital loss of $p(\omega)\, \Pi_f(\omega)$ for $\omega \in E^U - E^I$.[9] Hence, it is apparent that a firm which maximizes stock market value will not insure against losses in E^U.[10]

The Stock, Bond, and Insurance Markets

Now, introduce risky debt instruments as well as insurance contracts. Suppose the firm has issued bonds for which it promises to repay B_f dollars at the end of the period if the firm's earnings are sufficient. Suppose that the firm's debt is not secured and that the firm's losses are priority claims (i.e., the losses are paid before the bondholders receive payment). Then the return to all bondholders is $\min\{B_f, \max\{\Pi_f, 0\}\}$. The return per share is $\min\{B_f, \max\{\Pi_f, 0\}\}/B_f = \min\{1, \max\{\Pi_f, 0\}/B_f\}$ and so letting r_f denote the share price of corporation f's debt, consumption now and then may be expressed as

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[9] The payoff on the insurance decision in state ω is $[p(\omega) - q(\omega)]\, y_f\,(\omega) = 0$.

[10] Although the corporate motive for purchasing insurance is being considered here, it is apparent that if the firm is run by a manager whose compensation depends on the solvency of the firm then that manager has an incentive to purchase insurance and reduce the size of the insolvency set. The manager may also have the incentive to insure against the event E^U. Of course, this motivation is quite clearly based on risk aversion. There is clearly an agency cost in this case.

$$c_{i0} = m_{i0} - \Sigma_\Omega\, p(\omega)x_i(\omega) - \Sigma_\Omega\, q(\omega)y_i(\omega) - r_f z_{if} \tag{7}$$

and

$$c_{i1}(\omega) = m_{i1} + x_i(\omega) + y_i(\omega) + z_{if} \min\{1, \max\{\Pi_f(\omega), 0\}/B_f\} \tag{8}$$

where z_{if} is the number of shares of corporation f's debt held by individual i. Given the consumption now and then specified in (7) and (8), the consumer's problem is the same as that stated in (P). The only alteration required in the analysis is the specification of the condition for an optimal debt purchase, given in (9).

$$\Sigma_\Omega[-D_1u_i\, r_f + D_2u_i \min\{1, \max\{0, \frac{\Pi_f(\omega)}{B_f}\}]\psi(\omega) = 0. \tag{9}$$

Let $\Omega - B^U = \{\omega \in \Omega \mid \min\{1, \max\{0, \Pi\}/B_f\} = 1\}$ and $\Omega - E^U = \{\omega \in \Omega \mid \max\{0, \Pi_f(\omega)\}/B_f = \Pi_f(\omega)/B_f\}$, then first order condition (9) may be rewritten as

$$r_f = \frac{\Sigma_{\Omega - B^U}\, D_2u_i\, \psi(\omega)}{\Sigma_\Omega\, D_1u_i\, \psi(\omega)} + \frac{\Sigma_{B^U - E^U}\, D_2u_i\, \frac{\Pi_f(\omega)}{B_f}\, \psi(\omega)}{\Sigma_\Omega\, D_1u_i\, \psi(\omega)}$$

$$= \Sigma_{\Omega - B^U}\, p(\omega) + \Sigma_{B^U - E^U}\, p(\omega)\, \frac{\Pi_f(\omega)}{B_f}$$

The second equality follows due to (2). Notice that the total market value of the firm's risky debt is $D_f^U = r_f\, B_f$ and so

$$D_f^U = \Sigma_{\Omega - B^U}\, p(\omega)\, B_f + \Sigma_{B^U - E^U}\, p(\omega)\, \Pi_f(\omega) \tag{10}$$

This result is intuitively appealing because one bond is equivalent to a portfolio of stock with a one dollar payoff for all $\omega \in \Omega - B^U$, with a fraction $\Pi_f(\omega)/B_f$ of a dollar payoff for $\omega \in B^U - E^U$, and with a zero dollar payoff otherwise. Letting S_f^U denote the value of equity, note that

$$S_f^U = \Sigma_\Omega\, p(\omega) \max\{0, \Pi_f(\omega) - B_f\}$$

$$= \Sigma_{\Omega - B^U}\, p(\omega)\, [\Pi_f(\omega) - B_f]. \tag{11}$$

Then the value of the firm is $V_f^U = S_f^U + D_f^U$ or

$$V_f^U = \Sigma_{\Omega - E^U}\, p(\omega)\, \Pi_f(\omega). \tag{12}$$

Of course, it may be noted that this is the 1958 Modigliani-Miller capital structure irrelevance result in the complete market context with costless bankruptcy.

Next, consider the value of the insured firm (i.e., $V_f^I = S_f^I + D_f^I$). Using the same method, as that above, it follows that

$$V_f^I = \Sigma_{\Omega - E^I}\, p(\omega)\, \Pi_f(\omega)$$

and so $V_f^I \leq V_f^U$. Hence the presence of risky debt does not alter the conclusions as long as bankruptcy is costless as assumed here. Of course, it should also be noted that if the firm purchases insurance against states in $\Omega - E^U$ then, although it does not increase the total firm value, it does increase the value of the firm's debt. Hence insurance may be used to shift risk from bondholders to stockholders. This point will be important in the discussion of agency costs.

Costly Bankruptcy

In this section, the assumption that the unlevered uninsured firm may become insolvent with some positive probability (i.e., $P(E^U) > 0$) is maintained. Similarly, as above, let $B^U \supseteq E^U$ denote the bankruptcy set of the levered uninsured firm. However, now suppose that there is a cost $c > 0$ associated with the bankruptcy event. The presence of a transaction cost should provide no incentive to insure for the unlevered firm but some incentive for the levered firm.[11]

The market value of the levered uninsured firm is the value of its equity plus the value of its debt (i.e., $V_f^U = S_f^U + D_f^U$). The value of the levered uninsured firm's stock is

$$S_f^U = \Sigma_{\Omega - B^U}\, p(\omega)\, [\Pi_f(\omega) - B_f],$$

while the value of the levered uninsured firm's debt, given costly bankruptcy is

$$D_f^U = \Sigma_{\Omega - B^U}\, p(\omega) B_f + \Sigma_{B^U - E^U}\, p(\omega)\, [\Pi_f(\omega) - c].^{12} \tag{13}$$

It follows that the total value of the firm is

$$\begin{aligned} V_f^U &= \Sigma_{\Omega - B^U}\, p(\omega)\, \Pi_f(\omega) + \Sigma_{B^U - E^U}\, p(\omega)\, [\Pi_f(\omega) - c] \\ &= \Sigma_{\Omega - E^U}\, p(\omega)\, \Pi_f(\omega) - \Sigma_{B^U - E^U}\, p(\omega)\, c \end{aligned} \tag{14}$$

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[11] The lack of incentive in the unlevered case follows simply by noting that stockholders received max $\{0, \Pi_f\}$ and so the transaction cost has no effect on the payoffs they receive.

[12] Given limited liability, the bondholders actually receive $\max\{0, \Pi_f - c\}$ in the event $B^U - E^U$. To simplify the analysis here suppose that the bankruptcy cost is sufficiently small relative to the net corporate earnings so that $\max\{0, \Pi_f - c\} = \Pi_f - c$ for all $\omega \in B^U - E^U$.

The last term on the right-hand side of (14) is the present value of the bankruptcy costs. It may be noted that the 1958 Modigliani-Miller [11] theorem holds here if either the bankruptcy cost is zero or the bankruptcy set of the levered firm equals that of the unlevered firm (i.e., $c = 0$ or $B^U = E^U$); otherwise, the firm's capital structure is relevant. Of course, this value does not incorporate insurance and, although it may not be included in the literature, it seems apparent that insurance allows the corporation to avoid some bankruptcy costs.

Now consider the value of the insured firm. Suppose both the uninsured and insured firms have the same debt issue B_f. The bankruptcy sets of the uninsured and insured firms are B^U and B^I, respectively. In addition it may be noted that $B^I \subseteq B^U$. Then the value of the levered insured firm is

$$V_f^I = \Sigma_{\Omega - E^I}\, p(\omega)\, \Pi_f(\omega) - \Sigma_{B^I - E^I}\, p(\omega)\, c \qquad (15)$$

With some qualification, it now follows that the value of the levered uninsured corporation is less than the value of the unlevered uninsured corporation. The qualification is that the firm not insure against states in E^U; as was shown in the previous section, any insurance against states in E^U yields a capital loss to the firm's financial asset holders. If the insured firm does not insure against the insolvency states then $E^I = E^U$ and the difference in corporate values is

$$\begin{aligned} V_f^I - V_f^U &= \Sigma_{B^U} - E^U\, p(\omega)\, c - \Sigma_{B^I - E^U}\, p(\omega)\, c \\ &= \Sigma_{B^U - B^I}\, p(\omega)\, c > 0, \end{aligned}$$

for B^I such that $E^U \subseteq B^I \subset B^U$. The increase in value, due to insurance, is simply the present value of the saving in bankruptcy costs.

It should also be observed that the insurance decision increases the value of the corporation's debt (i.e., D_f^I). Since

$$D_f^I = \Sigma_{\Omega - B^I}\, p(\omega)\, B_f + \Sigma_{B^I - E^U}\, p(\omega)\, [\Pi_f(\omega) - c],$$

the result follows by noting that increasing insurance coverage increases the size of the solvency set, equivalently, the non-bankruptcy set $\Omega - B^I$. The value of the firm's debt is maximized when the firm selects insurance coverage such that $B^I = E^U$ This insurance coverage will not eliminate bankruptcy but it will increase the security of the debt. For this case in which $B^I = E^U$, the increase in the value of the debt issue is

$$D_f^I - D_f^U = \Sigma_{B^U} - E^U\, p(\omega)\, B_f - \Sigma_{B^U} - E^U\, p(\omega)\, [\Pi_f(\omega) - c] > 0.$$

The difference is positive because $B_f > \Pi_f$ in the event $B^U - E^U$. Of course, a bond covenant may be necessary to achieve this insurance coverage since it will not increase the stock value of the firm. To see this, note that

$$S_f^I - S_f^U = \Sigma_{B^U - E^U}\, p(\omega)\, [\Pi_f(\omega) - B_f] < 0,$$

Since $B_f > \Pi_f$ in the event $B^U - E^U$.

Agency Problems

In this section, the use of insurance contracts in resolving some conflict of interests problems (i.e., encountered by the corporate management in dealing with bondholders) are analyzed. Since the corporate manager also represents the interests of stockholders, there is a potential for conflict between the manager and bondholders, or equivalently, between the manager and the bondholders' trustee. This will be the case if it is possible for the manager to take actions which benefit one group and are detrimental to the other. If the bonds represent secured debt then there is no conflict. If not, then an agency problem may exist.

The agency relationship can be thought of as a contract between the principal (i.e., the bondholders' trustee)[13] and an agent (i.e., the corporate manager). The agent acts in behalf of the principal. The contract specifies the bounds on the actions that may be taken by the agent. If the contract covers all possible contingencies then there is no real delegation of authority and so no agency problem. If the contract is incomplete so that the agent has some discretion in the selection of actions then there is at least the potential for a conflict of interests. The conflict occurs because both the principal and the agent behave in accordance with their own self interests. The principal can limit the divergence of interests by providing provisions in the contract which give the agent the appropriate incentives to act in the principal's interest; in addition, the principal can monitor the activity of the agent. However it is not usually possible to specify the contract in such a way as to completely eliminate the conflict of interests problem. Hence, it will usually be the case that there is a difference between the action taken by the agent and the action which is in the best interests of the principal. The agency cost is defined, by Jensen and Meckling [7], as the sum of the monitoring expenditures of the principal, the bonding expenditures of the agent, and the residual loss; this residual loss is the loss in the market value of the corporation.[14]

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[13] The legal trustee for the bondholders may be treated as the single principal. It should be added that the trustee acts on behalf of the bondholders. The trustee's problem is the selection of bond covenants which limit the divergence of interests between corporate management and the bondholders. In general, the trustee may have a problem in selecting covenants which provide a solution to the conflict because of the different risk aversion measures of the bondholders. In the two cases considered here, however, the bondholders will unanimously support a covenant which provides management with the incentive to maximize the risk adjusted net present value of the corporation. It should also be noted that in general there may be an agency problem between the trustee and bondholders (i.e., between the agent and the principals). In the cases considered here that problem does not arise because of the unanimity.

[14] Jensen and Meckling [7] also define the residual loss as the dollar equivalent of the loss in expected utility experienced by the principal. Although this notion of residual loss is measurable for a particular principal, this definition poses problems when a trustee represents many principals because the residual loss of any bondholder will depend on the bondholder's measure of risk aversion and on the proportion of the contract owned.

Underinvestment

The first agency problem considered here is encountered by the manager in making investment decisions. Jensen and Smith [8] noted that one source of conflict is underinvestment. They observed that

> . . . when a substantial portion of the value of the firm is composed of future investment opportunities, a firm with outstanding risky bonds can have incentives to reject positive net present value projects if the benefit from accepting the project accrues to the bondholders [8, p. 111].[15]

The incentive need not be so extreme that it causes the manager to reject a project; the manager may under invest by limiting the size of the project. Suppose the earning of firm f is $\Pi_f(I_f, \omega)$ where

$$\Pi_f(I_f, \omega) = R_f(I_f, \omega) - L_f(I_f, \omega)$$

$D_1\Pi_f > 0$ and $D_{11}\Pi_f < 0$, so that the corporate payoff is increasing at a decreasing rate in the dollar investment level I_f. Also suppose that the corporation has issued risky debt. As above, let B^U denote the firm's bankruptcy set. Then, with no corporate taxes and no bankruptcy cost, the stock market value of the firm is S_f^U where

$$S_f^U = \Sigma_{\Omega - B^U}\, p(\omega)\, [\Pi_f(I_f, \omega) - B_f],$$

where $\Omega - B^U = \{\omega \in \Omega \mid \Pi_f(I_f, \omega) - B_f \geq 0\}$. The market value of the corporation's risky debt is D_f^U where

$$D_f^U = \Sigma_{\Omega - B^U}\, p(\omega)\, B_f + \Sigma_{B^U - E^U}\, p(\omega)\, \Pi_f(I_f, \omega).$$

Therefore the total value of the corporation is

$$V_f^U = S_f^U + D_f^U = \Sigma_{\Omega - E^U}\, p(\omega)\, \Pi_f(I_f,\omega).$$

Now, consider the investment levels selected by the corporate manager who selects I_f to maximize net present value for all corporate claimholders versus for all stockholders (i.e., V_f^U - I_f versus S_f^U - I_f). Maximizing the net present value for all claimholders yields the condition

$$\Sigma_{\Omega - E^U}\, p(\omega)\, D_1\Pi_f(I_f, \omega) - 1 = 0 \tag{16}$$

Let I_f^V denote the investment level implicitly defined by condition (16). Similarly, maximizing the net present value for stockholders yields the condition

$$\Sigma_{\Omega - B^U}\, p(\omega)\, D_1\Pi_f(I_f, \omega) - 1 = 0. \tag{17}$$

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[15] Also see [2, 12, 14].

The derivative in (17) is only appropriate if the change in the investment level does not change the bankruptcy set B^U. Let I_f^S denote the investment level implicitly defined by condition (17). Now, it is clear that if I_f^S is substituted into the derivative on the left-hand side of (16) then the derivative is positive. Therefore, $I_f^V > I_f^S$. The firm which maximizes the net present value for stockholders will not invest as much because some of the additional value accrues to the bondholders rather than the stockholders. The size of the agency cost if $V_f^U(I_f^V) - V_f^U(I_f^S) > 0$.

Now, consider how a bond covenant, requiring insurance, could be structured to reduce or even eliminate this agency problem. If the firm is required to purchase enough insurance to make $B^I = E^U$ then the firm maximizing the net stock value $S_f^I - I_f$ will select $I_f^S = I_f^V$, in which case the agency problem is eliminated. Recall that the solvency set is $\Omega - B^U = \{\omega \in \Omega \mid \Pi_f(I_f, \omega) - B_f \geq 0\}$. Hence if $y_f(\omega) \geq 0$ is the minimum amount such that

$$\Pi_f(I_f^V, \omega) + y_f(\omega) - B_f \geq 0, \tag{18}$$

for $\omega \in \Omega - E^U$, then $B^I = E^U$. Notice that this insurance provision does not require full insurance but it does require the insurance coverage to be tied to the size of the debt issue. It may be noted that this insurance coverage $y_f(\omega)$ is a nondecreasing function of B_f and that $y_f(\omega)$ is zero for those states of nature such that $\Pi_f(I_f^V, \omega) - B_f > 0$. If this insurance scheme is used then the corporate debt is safer and its value is greater than the debt value of the uninsured corporation which invests I_f^S. Similarly, the stock market value of the insured corporation is greater than the stock market value of the uninsured corporation which invests I_f^S. To see this, note that

$$\begin{aligned} D_f^I &= \Sigma_{\Omega - E^U}\, p(\omega)\, B_f \\ &> \Sigma_{\Omega - B^U}\, p(\omega)\, B_f + \Sigma_{B^U - E^U}\, p(\omega)\, \Pi_f(I_f, \omega) \\ &= D_f^U, \end{aligned}$$

and

$$\begin{aligned} S_f^I &= \Sigma_{\Omega - E^U}\, p(\omega)\, [\Pi_f(I_f^V, \omega) - B_f] \\ &> \Sigma_{\Omega - B^U}\, p(\omega)\, [\Pi_f(I_f^S, \omega) - B_f] \\ &= S_f^U. \end{aligned}$$

Therefore all claimholders can be made better off by this insurance scheme and, of course, this insurance scheme eliminates the agency cost of debt.

Finally, note that the insurance coverage implicitly specified in (18) may not always be feasible. If the insurance coverage in state ω is $y_f(\omega) > L_f(\omega)$ then it is not feasible. In this case, an insurance covenant requiring full insurance in some states will reduce but not eliminate the agency problem.

Asset Substitution

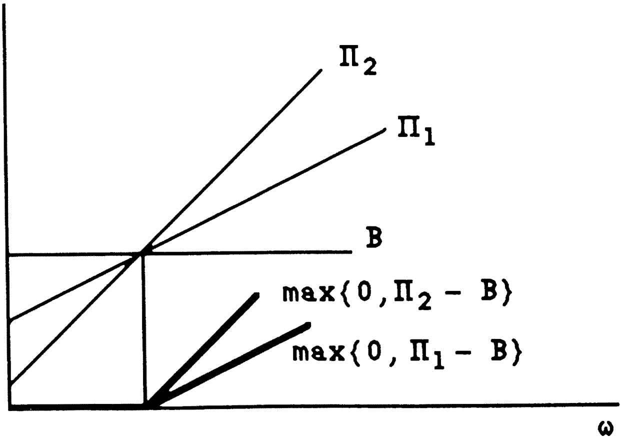
The second agency problem considered here is encountered by the corporation in selecting among mutually exclusive investment projects. Jensen and Smith noted that

> . . . the value of the stockholders' equity rises and the value of the bondholders' claim is reduced when the firm substitutes high risk for low risk projects [8, p. 111].[16]

Of course, bondholders are aware of this possibility (i.e., this attempt to shift risk) and so it is reflected in a lower value for the corporation's debt issue. An insurance scheme is constructed here that will reduce or eliminate the risk-shifting incentive and so another source of the agency cost of debt.

In order to demonstrate the agency problem, suppose the corporation is considering two mutually exclusive investment projects. Call them projects one and two. Let $I_1 = I_2$ denote the dollar cost of the two projects and let Π_1 and Π_2 denote the random project earnings.[17] Suppose project earnings are positive for all states.[18] Suppose project two is riskier than project one, in the Rothschild-Stiglitz [13] sense;[19] in particular, let $\Pi_2 = (1+\theta)\Pi_1 - \theta E\Pi_1 - \theta$ $E\Pi_1$, where $\theta > 0$. Then Π_2 is a mean-preserving spread of Π_1, as shown in figure four (i.e., $\mu_2 = E\Pi_2 = E\Pi_1 = \mu_1$ and Π_2 has more weight in the tails of its distribution).[20]

Figure 4: Project Payoffs



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[16] Also see [5, 7] for similar statements.

[17] For notational simplicity in this section of the paper, the subscript indicating the corporation is dropped and replaced with a subscript indicating the project.

[18] The assumption $\Pi_j(\omega) > 0$ for all $\omega \in \Omega$ simply allows the result $V_j^I = V_j^U$, j = 1, 2, for any insurance scheme, to be used here.

[19] The random variable Π_f^2 is riskier than Π_f^1 if Π_f^2 equals Π_1 plus noise, or if every risk averter prefers Π_f^1 to Π_f^2, or if Π_f^2 has more weight in the tails of its distribution than Π_f^1.

[20] The state space is still assumed to be finite but it is easier to see the mean-preserving spread when Π_f is drawn as a continuous function of ω.

The first claim made in this section is that there exists a debt level B^* such that $S_2^U > S_1^U$ for $B > B^*$ and $S_2^U < S_1^U$ for $B < B^*$. This may be done simply by showing that in the unlevered case $S_1^U = V_1^U > V_2^U = S_2^U$ and in the sufficiently highly levered case $S_2^U > S_1^U$. Then the Intermediate Value Theorem yields the existence of a B^*, since $S_1^U - S_2^U$ is a continuous function of B.

Following the outlined procedure, first, it must be shown that if the corporation is unlevered and if project two is riskier than project one, then $S_1^U = V_1^U > V_2^U = S_2^U$. Note that the difference in value is

$$\begin{aligned} V_1^U - V_2^U &= \Sigma_\Omega\, p(\omega)\, [\Pi_1(\omega) - \Pi_2(\omega)] \\ &= \Sigma_\Omega\, k(\omega)\, [\Pi_1(\omega) - \Pi_2(\omega)]\, \psi(\omega), \qquad (19) \end{aligned}$$

where $k(\omega) \equiv p(\omega)/\psi(\omega)$ is the price of a one dollar payoff contingent on state ω occurring and it is decreasing in ω.[21] Substituting $(1+\theta)\,\Pi_1 - \theta\, E\Pi_1$ for Π_2, it follows that (19) may be equivalently expressed as

$$\begin{aligned} V_1^U - V_2^U &= \Sigma_\Omega\, k(\omega)\, \theta\, [E\Pi_1 - \Pi_1(\omega)]\, \psi(\omega) \\ &= Ek\, \theta\, [E\Pi_1 - E\Pi_1] - \theta\, \mathrm{Cov}(k, \Pi_1) \\ &= -\theta\, \mathrm{Cov}(k, \Pi_1) \qquad (20) \\ &> 0. \end{aligned}$$

Note that k decreasing and Π_1 increasing in ω make the covariance term negative and so the whole term positive. Therefore, given no debt and no insurance, the stock market value of project one exceeds the stock market value of project two (i.e., $V_1^U - V_2^U = S_1^U - S_2^U > 0$).

Second, it must be shown that in the highly levered case the stock market value of the riskier project is greater (i.e., $S_2^L > S_1^L$). To do this simply note that for B sufficiently large $\max\{0, \Pi_2 - B\} \geq \max\{0, \Pi_1 - B\}$ for all $\omega \in \Omega$ and strictly greater for some ω, as shown in figure five. Then it follows trivially that

$$\begin{aligned} S_2^L &= \Sigma_\Omega\, p(\omega)\, \max\{0, \Pi_2(\omega) - B\} \\ &> \Sigma_\Omega\, p(\omega)\, \max\{0, \Pi_1(\omega) - B\} \\ &= S_1^L. \end{aligned}$$

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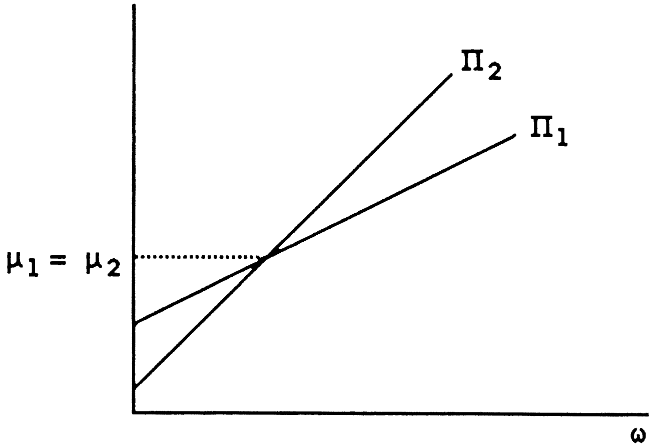
[21] This follows because all corporate payoffs and so aggregate wealth increase in ω. See [9, 15] for a discussion of this result.

Figure 5: Projects' Equity Payoffs



Then continuity completes the proof and a debt level B* exists such that $S_2^L > S_1^L$ for $B > B^*$ and $S_2^L < S_1^L$ for $B < B^*$. Therefore, in the absence of any mechanism to avoid the agency problem, the corporation with a debt level $B > B^*$ has an incentive to accept the riskier project. The agency cost of debt, in this case is

$$V_1^U - V_2^U = \Sigma_\Omega\, p(\omega)\, [\Pi_1(\omega) - \Pi_2(\omega)] > 0.$$

Now, consider whether a bond covenant requiring insurance can be written in such a way that even highly levered corporations have an incentive to select the less risky project. The less risky project is preferred by bondholders because the debt issue is safer. Of course, insurance makes the debt issue safer regardless of project selection and so limits the ability of the corporation to shift risk to the bondholders. To see this, note that the stock market value of the insured project $j = 1, 2$, is

$$S_j^I = \Sigma_\Omega\, p(\omega) \max\{0, \Pi_j(\omega) + y(\omega) - B\} - \Sigma_\Omega\, q(\omega)\, y(\omega) \qquad (21)$$

and the debt value is

$$D_j^I = \Sigma_\Omega\, p(\omega) \min\{B, \Pi_j(\omega) + y(\omega)\} \qquad (22)$$

It follows that the total value of project j is

$$V_j^I = \Sigma_\Omega\, p(\omega)\, \Pi_j(\omega)$$

Therefore the insurance decision will not affect the total value of the corporation. Project selection, however, will affect the total value of the corporation. What is important here is that the insurance decision will affect how the earnings are allocated to bondholders and stockholders. From (21) and (22), it is apparent that insurance yields a transfer of payoff from stockholders to bondholders. If the transfer is sufficiently large then the corporation will not be able to shift enough risk to bondholders to make the selection of the riskier project optimal.

In order to construct the insurance scheme, suppose the corporate has a debt issue with $B > B^*$. Then the uninsured firm has the incentive to select riskier project two. Now, suppose an insurance covenant is imposed which requires uniform coverage (i.e., $y(\omega) = c$ for all $\omega \in \Omega$). Then the difference in the stock values of the two projects is

$$S_1{}^I - S_2{}^I = \Sigma_\Omega\, p(\omega) \max\{0,\, \Pi_1(\omega) + c - B\}$$
$$- \Sigma_\Omega\, p(\omega) \max\{0,\, \Pi_2(\omega) + c - B\}.$$

The cost of the insurance is the same regardless of project selection and so has no effect. More importantly, notice that if the uniform coverage c is selected so that $B - c < B^*$, or equivalently, so that $c > B - B^*$, then the stock market value of insured project one exceeds the stock market value of insured project two. This insurance is equivalent to a reduction in the promised debt repayment and so the same analysis as that above yields $S_1{}^I > S_2{}^I$.

This insurance scheme is only one of a variety of insurance provisions that may reduce or eliminate the agency cost. The uniform coverage scheme has the advantage of being a particularly simple contract. As long as $c \leq L$ for all states, the uniform coverage plan will be feasible. Finally, it should also be noted that the plan does not require full coverage of losses but it does require coverage to increase with the size of the debt issue.

Conclusion

The notion of risk management implies that the corporation plays an active role in reducing the riskiness of the corporate payoff in much the same way that an individual investor would reduce risk by diversifying his portfolio. The analysis here shows that it is not always necessary for the corporation to actively pursue any risk reduction policy (i.e., risk management is irrelevant). This is the case if the corporation's risk management operation does not affect the payoffs which investors can achieve by diversifying their own portfolios. The first and most naive version of the financial markets model demonstrates this case.

The corporation has an active role to play if it is possible for the corporation to alter the payoff distribution in a way which investors cannot duplicate on personal account. In the sections on costly bankruptcy and agency problems, the analysis shows that minor modifications of the financial markets model provide the corporation with an incentive to purchase insurance (i.e., $V_f{}^I > V_f{}^U$). Two results emerge from the analysis. The first is that $V_f{}^I \geq V_f{}^U$ in an economy with costly bankruptcy and it is a strict inequality if the uninsured unlevered firm has a positive probability of bankruptcy. This is the case because the firm can reduce the bankruptcy cost and this is something which individual investors cannot achieve on personal account. This result is not substantially altered by the introduction of a corporate tax and risky debt. The second result is that insurance may be used to reduce or eliminate some of the agency costs of debt. The corporate manager who selects the investment level preferred by stockholders can alleviate the underinvestment problem by partially insuring against bankruptcy and thereby increasing the stock market value and bond market value of the corporation. This agency cost is like a deadweight loss; in the process of eliminating it the manager can make all the corporate claimholders better off. The asset substitution problem can be solved by ensuring that the corporation has some minimal level of insurance. If so, the manager will find it in the interests of the stockholders to select the asset which is also preferred by bondholders.

Finally, some of the results contain some interesting implications for a more robust theory of financial markets. With the introduction of agency problems, the corporation may reduce or eliminate the agency cost, equivalently, the deadweight loss, by appropriate insurance provisions. For example, in the case of asset substitution, the analysis shows that there is a minimal level of uniform coverage which eliminates the conflict of interests between stockholders and bondholders. This result shows that the corporation's capital structure and insurance decisions are interdependent because a larger corporate debt issue requires a larger level of insurance coverage to eliminate the conflict.

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